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by

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Abstract

An expected lifetime-utility maximizing diet of junk and health food is analyzed. The stationary junk-food consumption level is equal to the ratio of the recovery capacity of a perfectly healthy person to the sensitivity of her health to junk food. The greater the difference between the relative taste and the stationary relative price of junk food, rate of time preference, and elasticity of satisfaction from food, the better the stationary health of the rational junk-food consumer. The greater the full capacity income, recovery capacity, and health sensitivity to junk-food, the worse the stationary health of the rational junk-food consumer.

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Food can be classified in accordance with its fat, sugar and salt contents as junk or healthy. Because of its high concentration of sugar, fat and salt, junk food is often tastier than its low calories, leaner and less salty substitute. Due to its relatively expensive ingredients, health food is often more expensive than junk food. For instance, diet ice cream is relatively expensive because of its synthetic sweetening and creaming inputs. Yet, for many consumers, diet ice cream is not as tasty as its buttery and sugary rival.

The short-term taste and price advantages of junk food are, at least, partially offset by the long-term adverse effects of junk food on health and life expectancy. Rational food consumers are not myopic - they are aware of the short-term advantages and long-term disadvantages of junk-food consumption. In addition to the taste and price differentials, rational consumers incorporate the risk differential into an expected lifetime utility maximization analysis of the composition of junk-food and health-food products in their diet.

Taste, price and risk differences are not exclusive to junk-food products and their healthier substitutes. They may also provide an explanation to decisions on the consumption of commodities such as coffee, tea, beer and self-rolled cigarettes. The comparison of the taste, price and health impeding effects of coffee, tea, beer and self-rolled cigarettes to those of their healthier substitutes (decaffeinated coffee, herbal tea, light beer and filter cigarettes, respectively) within a lifetime utility maximization framework with uncertain life expectancy constitutes a complementary approach to the rational addiction model proposed by Gary Becker and Kevin Murphy (1988) and applied by Frank Chaloupka (1991), Gary Becker, Michael Grossman and Kevin Murphy (1994), Nilss Olekalns and Peter Bardsley (1996), Michael Grossman, Frank Chaloupka and Ismail Sirtalan (1998) and many others to the consumption of cigarettes, alcohol and coffee.

Consistently with Karen Dynan's (2000) empirical findings with panel household data, the present analysis assumes that food consumption is neither addictive nor a formed habit. An intertemporal consumer model incorporating the taste, price and risk differences between junk food and its healthier substitute is proposed. The building blocks of the model generating a rational choice of a diet of junk food and health food are presented in section I. Similar to Amnon Levy (2000, 2002a and 2002b), life expectancy is taken to be random, the probability of dying is related to health and age, and rational behaviour is defined as expected lifetime-utility maximization.

The expected lifetime-utility maximization problem is presented in section II and the properties of the rational diet of junk-food and value of health are discussed in section III. It is shown that at every instance a rational junk-food consumer maintains an equality between the marginal satisfaction from junk-food consumption, discounted by her time preference and prospects of survival, and the value of the marginal damage to her health. Along her rational junk-food consumption path, the rate of change of the shadow value of health is negative and linearly diminishing in junk-food consumption and health. These adverse effects of junk-food consumption and health on the rate of change of the shadow value of health are amplified by the health sensitivity to junk food and moderated by the elasticity of the individual satisfaction from food consumption. The adverse effect of health on the rate of change of the shadow value of health is further amplified by the full capacity income, which can be attained by a perfectly healthy individual.

The rational long-run (stationary) consumption of junk food and the consumer's stationary health are presented in section V. It is shown that the rational stationary junk-food consumption level is equal to the ratio of the recovery capacity of a perfectly healthy person to the marginal adverse effect of junk-food consumption on health. The greater the difference between the relative taste and the stationary relative price of junk food, rate of time preference, and elasticity of satisfaction from food, the better the stationary health of the rational food consumer. In contrast, the greater her full capacity income, recovery capacity, and health sensitivity to junk-food, the worse the stationary health of the rational food consumer.

I. Building Blocks

The analysis employs the following notations:

t = a continuous time index, $t \in (0,T)$ where T is a positive scalar indicating the upper bound on a consumer's life expectancy;

$c_j(t)$ = the individual consumption of junk food at instance t ;

$c_h(t)$ = the individual consumption of health food at instance t ;

$x(t)$ = the individual health condition at instance t, a unit interval index $0 \le x(t) < 1$ with $x = 0$ representing a terminally ill person and $x = 1$ a perfectly healthy person;[1]

$p(t)$ = the junk food-health food price ratio;

$u(t)$ = the individual satisfaction from food at instance t;

a = the junk food-health food taste ratio;

$y(t)$ = the individual income at instance t;

$\hat{y}$ = a positive scalar indicating the full capacity income;

$f(t)$ = the probability of dying at instance t; and

$r(t)$ = the individual rate of time preference at instance t.

The building blocks of the rational junk-food consumption model are summarized by the following eight assumptions.

Assumption 1 (relative price): Junk food is cheaper than health food. That is, $p(t) < 1$.

Assumption 2 (instantaneous satisfaction): The individual instantaneous satisfaction from eating is represented by a utility function $u(c_j(t), c_h(t))$ having the following properties. Food is essential -- $u(0,0) = 0$. However, neither junk food nor health food is essential -- $u(0, c_h) > 0 < u(c_j, 0)$. The marginal satisfaction with respect to each type of food is positive and diminishing -- $u_j, u_h > 0$, $u_{jj}, u_{hh} < 0$.

Assumption 3 (relative taste): Junk food is tastier than health food. For equal intakes, the marginal satisfaction of junk food is higher than that of health food. That is, $u_j > u_h$ for every $c_j \le c_h$.

Consistently with assumptions 2 and 3, the following explicit utility function is considered

$$u(t) = [ac_j(t) + c_h(t)]^b \quad (1)$$

where $a > 1$ is the relative taste coefficient and $0 < b < 1$ is the elasticity of the individual satisfaction from food.

Assumption 4 (instantaneous income): The ratio of the individual instantaneous income to the full capacity income is equal to the individual instantaneous health condition. In other words,

$$y(t) = x(t)\hat{y} \quad (2)$$

revealing that the full capacity income could only be attained by a perfectly healthy individual, and that the income of a terminally ill person is nil.

Assumption 5 (instantaneous budget constraint): There is no borrowing or lending (for simplicity sake) and the individual instantaneous income is fully spent on buying junk food and health food. Taking the price of health food as a numeraire, the budget constraint is given by

$$p(t)c_j(t) + c_h(t) = x(t)\hat{y}\,. \quad (3)$$

Assumption 6 (health): The individual health is deteriorated by eating junk food and improved by a natural recovery process. Health-food only helps maintaining the individual health at the same level.[2] Correspondingly, the instantaneous change in the individual health can be displayed by the following motion equation

$$\dot{x}(t) = [g - dc_j(t)]x(t) \quad (4)$$

where, g is a positive scalar indicating the recovery capacity of a perfectly healthy person (for whom $x = 1$) and d is a positive scalar indicating the marginal adverse effect of junk-food consumption on the rate of change of the individual health. Loosely interpreted, d is the individual health sensitivity to junk food.

Assumption 7 (survival probability): Let $F(t)$ be the cumulative distribution function associated with the probability of dying $f(t)$, then $1-F(t)$ indicates the probability of living beyond t (i.e., survival at t). This probability is assumed to decline with the individual age and to rise with her health. It converges to zero either when the individual age approaches the upper-bound life expectancy (T), when the individual health is completely deteriorated ($x=0$), or both. That is,

$$1-F(t)=[1-e^{-m(t)(T-t)}]x(t)\,, \qquad m>0\,. \tag{5}$$

The positive scalar m can be interpreted as the individual youth retaining capacity.[3]

Assumption 8 (time-consistent preferences): The individual rate of time preference is positive and time invariant. That is, $r(t)=r$ for every $t\in(0,T)$.

II. Rational Choice

It is postulated that rational individuals chose their junk and health food diet path so as to maximise their expected lifetime satisfaction from food subject to their health motion equation. Since life expectancy is random, expected-lifetime-satisfaction-maximising food consumers multiply their accumulated satisfaction from food between the starting point of their planning horizon, 0, to their possible time of death t (i.e., multiply $\int_0^t e^{-rt}u(t)dt$) by the probability of dying at time t (i.e., $f(t)$). The products of $f(t)$ and $\int_0^t e^{-rt}u(t)dt$ associated with any possible life expectancy $0\le t\le T$ are considered by such rational consumers. The sum of all these products is these consumers' expected lifetime-satisfaction from food. It is given by the following double-integral expression

$$V = \int_0^T f(t) \int_0^t e^{-rt} u(t) dt dt \, . \tag{6}$$

Integrating by parts, this expected lifetime-satisfaction can be equivalently rendered by a mathematically more manageable single-integral expression:

$$V = \int_0^T [1 - F(t)] e^{-rt} u(t)) dt \, . \tag{7}$$

A detailed mathematical explanation is given in the Appendix.

The analysis of the rational diet trajectory can be further simplified by expressing c_h as a function of c_j. Recalling the instantaneous budget constraint,

$$c_h(t) = x(t)\hat{y} - p(t)c_j(t) \, . \tag{8}$$

The substitution of Eq. (8) into Eq. (1) renders the instantaneous satisfaction function as

$$u(t) = \Big[[a - p(t)] c_j(t) + x(t)\hat{y} \Big]^b \, . \tag{9}$$

By virtue of assumptions 1 and 3, the difference between the relative taste and the relative price is positive (i.e., $a - p(t) > 0$). Hence, the marginal instantaneous satisfaction from junk food, in this concentrated form, is positive and diminishing. In turn, V is concave in the control variable c_j.

By substituting Eq. (5) and Eq. (9) into Eq. (7) for $1 - F(t)$ and $u(t)$, respectively, the individual expected lifetime-satisfaction maximizing junk-food consumption path is now found by

$$\max_{\{c_j\}} \int_0^T [1-e^{-m(T-t)}]x(t)e^{-rt}\Big[[a-p(t)]c_j(t)+x(t)\hat{y}\Big]^b dt$$

subject to her health motion equation 4.

III. Properties of the Rational Junk-Food Consumption and Health

The present-value Hamiltonian corresponding to the aforementioned constrained maximization problem is

$$H(t) = \underbrace{[1-e^{-m(T-t)}]}_{\Phi}x(t)e^{-rt}\underbrace{[(a-p)c_j(t)+x(t)\hat{y}]}_{Z}{}^b + l(t)\underbrace{[g-dc_j(t)]x(t)}_{\dot{x}} \qquad (10)$$

where the costate variable $l(t)$ indicates the shadow present value of the individual health at t. Since $0 < b < 1$ and $a - p(t) > 0$, H is concave in both the control variable (c_j) and the state variable (x) and hence, in addition to the state equation (Eq. (4)), the following conditions are necessary and sufficient for maximum expected lifetime satisfaction from junk-food consumption:

$$\dot{l} = -\frac{\P H(t)}{\P x(t)} = -\Phi(t)e^{-rt}\underbrace{Z(t)^b}_{u(t)} - \Phi(t)x(t)e^{-rt}\underbrace{bZ(t)^{b-1}\hat{y}}_{u_x(t)} + l d c_j(t) \qquad (11.1)$$

$$\frac{\P H(t)}{\P c_j} = \Phi(t)x(t)e^{-rt}\underbrace{bZ(t)^{b-1}[a-p(t)]}_{u_j(t)} - l d x(t) = 0\,. \qquad (11.2)$$

The optimality condition, Eq. (11.2), requires equality between the marginal satisfaction from junk-food consumption after being discounted by both the individual time preference and prospects of survival (the first term on the left-hand side) and the value of the marginal damage to the individual health caused by the junk-food consumption (the second term on the left-hand side) at every instance. This optimality condition and the adjoint Eq. (11.1), also imply that along the individual optimal junk-food consumption path the rate of change of the shadow value of health is negative and linearly eroded by junk-food consumption and health:

$$\frac{\dot{l}(t)}{l(t)} = -\left(\frac{d(1-b)}{b}\right)c_j(t) - \left(\frac{d(1+b)\hat{y}}{b(a-p)}\right)x(t)\,. \tag{12}$$

The effect of junk food on the individual valuation of health is clear. The rationale for the negative effect of health on the shadow value of health is that when health is more abundant its value for the individual is less important. That is, people attach a higher value to health when their health condition is bad than when their health condition is good. As can be seen from Eq. (12), the adverse effects of junk-food consumption and health on the rate of change of the shadow value of the individual health are amplified by the sensitivity of health to junk food (d) and moderated by the elasticity of the individual satisfaction from food consumption (b). The adverse effect of health on the rate of change of the shadow value of health is further amplified by the full capacity income ($\hat{y}$), which could be obtained by a perfectly healthy person.

IV. Rational Stationary Junk-Food Consumption and Health

By differentiating Eq. (11.2) with respect to time, substituting the right-hand sides of Eq. (11.1) and Eq. (11.2) for $\dot{l}$ and l, collecting terms and multiplying both sides of the resultant equation by $(1/x)e^{dt}$, the following singular control is obtained:

$$[(\dot{\Phi}(t)/\Phi(t)) - r - dc_j(t)]u_j(t) + [u_{jZ}(t)\dot{Z}(t) - u_Z(t)\dot{p}(t)] + d[u(t) + xu_x(t)] = 0\,. \tag{13}$$

The notion of steady state is used in the following to indicate possible long-run levels.[4]

In steady state (SS) $\dot{\Phi}$, $\dot{Z}$ and $\dot{p}$ are nil and consequently Eq. (13) is rendered as

$$-[r + dc_j^{ss}]u_j(Z^{ss}) + d[u(Z^{ss}) + x^{ss}u_x(Z^{ss})] = 0. \tag{14}$$

By substituting the explicit forms of u, u_j, u_x and Z into Eq. (14) and solving for x^{ss}, the rational stationary health is found to be an a-fine function of the rational stationary junk-food consumption level:

$$x^{ss} = \frac{rb(a - p^{ss})}{d\hat{y}(1-b)} - \left(\frac{a - p^{ss}}{\hat{y}}\right)c_j^{ss}. \tag{15}$$

Recalling Eq. (4), the rational stationary junk-food consumption level is equal to the ratio of the recovery capacity of a perfectly healthy person to the health sensitivity to junk food:

$$c_j^{ss} = g / d. \tag{16}$$

By substituting this ratio into Eq. (15) for c_j^{ss}, the rational stationary health condition can be rendered as

$$x^{ss} = \frac{(a - p^{ss})[rb - g(1-b)]}{(1-b)d\hat{y}}. \tag{17}$$

Thus, the stationary health condition of a rational consumer of junk food is better:

a. the larger the difference between the relative taste and the stationary relative price of junk food ($a - p^{ss}$),
b. the higher the rate of time preference (r), and
c. the greater the elasticity of satisfaction from food (b).

However, the stationary health condition of a rational consumer of junk food is worse:

d. the greater the full capacity income ($\hat{y}$),

e. the higher the recovery capacity (g), and

f. the greater the health sensitivity to junk food (d).

V. Conclusions

The analysis of rational consumption of junk food incorporated the short-term taste and price differences and the long-term risk difference between junk-food and health-food products into an expected lifetime-utility maximization framework. The analysis led to the following conclusions.

At every instance a rational junk-food consumer maintains an equality between the marginal satisfaction from junk-food consumption, discounted by her time preference and prospects of survival, and the value of the marginal damage to her health.

The rate of change of the shadow value of health is negative and linearly diminishing in junk-food consumption and health. The adverse effects of junk-food consumption and health on the rate of change of the shadow value of health are intensified by the sensitivity of health to junk food and moderated by the elasticity of satisfaction from food consumption. The adverse effect of health on the rate of change of the shadow value of health is further amplified by the full capacity income which could be attained by a perfectly healthy individual.

The rational stationary junk-food consumption level is equal to the ratio of the recovery capacity of a perfectly healthy person to the health sensitivity to junk food. The greater the difference between the relative taste and the stationary relative price of junk food, rate of time preference, and elasticity of satisfaction from food, the better the stationary health of the rational junk-food consumer. In contrast, the greater the full capacity income, recovery capacity, and sensitivity of health to junk food, the worse the stationary health of the rational junk-food consumer.

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Endnotes

1. The latter case is excluded -- "nobody is perfect".

2. Health-food fans may argue that, *ceteris paribus*, health food not only helps maintain personal health but also improves personal health. The incorporation of the latter assertion complicates the analysis and renders the model unsolvable.

3. An alternative specification, $1-F(t)=[1-e^{-mx(t)(t)(T-t)}]$, was considered. However, as this survival probability formula is convex in the individual health, the second-order conditions for maximum expected lifetime-utility are not satisfied.

4. The derivation of the stationary (or long run) levels of junk food consumption and health is consistent with the assumption that $T \to \infty$, in which case $\Phi = 1$ and the individual probability of survival is equal to her health condition. Namely, $1-F(t)=x(t)$.

APPENDIX: An explanation of the transition from Eq. (6) to Eq. (7)

$F(t)$ is the cumulative density function associated with the probability of dying at t (i.e., the probability of living up to t). Hence,

$$f(t) = F'(t) \tag{A1}$$

and Eq. (6) can be rendered as

$$J = \int_0^T F'(t)\left\{\int_0^t e^{-rt} u(t)dt\right\}dt = \int_0^T v(t)dU \tag{A2}$$

where,

$$v = \int_0^t e^{-rt} u(t)dt \tag{A3}$$

and

$$U = -(1 - F(t))\,. \tag{A4}$$

The integration by parts rule suggests that

$$J = \int_0^T vdU = Uv - \int_0^T Udv\,. \tag{A5}$$

Note, however, that

$$Uv = -\left[(1 - F(t))\int_0^t e^{-rt} u(t)dt\right]_0^T = 0 \tag{A6}$$

because when evaluated at the lower limit

$$Uv = -\left[(1 - F(0))\int_0^0 e^{-rt} u(t)dt\right] = 0 \tag{A7}$$

and when evaluated at the upper limit

$$Uv = -\left[(1-F(T))\int_0^T e^{-rt}u(t)dt\right] = 0 \qquad \text{(A8)}$$

as

$$F(T) = 1. \qquad \text{(A9)}$$

Hence,

$$J = -\int_0^T U dv\,. \qquad \text{(A10)}$$

By virtue of equation (A3)

$$dv = e^{-rt}dt \qquad \text{(A11)}$$

and the substitution of equations (A4) and (A11) into (A10) implies

$$J = \int_0^T e^{-rt}u(t)\Omega(t)dt \qquad \text{(A12)}$$

where

$$\Omega(t) \equiv -u(t) = 1 - F(t) \qquad \text{(A.13)}$$

and indicating the probability of living at least until t .